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Physics of W bosons at LEP

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Abstract

The high-energy and high-luminosity data-taking campaigns of the LEP e^+e^- collider provided the four collaborations, ALEPH, DELPHI, L3 and OPAL, with about 50 000 W-boson pairs and about a thousand singly-produced W bosons. This unique data sample has an unprecedented reach in probing some aspects of the Standard Model of the electroweak interactions, and this article reviews several achievements in the understanding of W-boson physics at LEP. The measurements of the cross sections for W-boson production are discussed, together with their implication on the existence of the coupling between Z and W bosons. The precision measurements of the magnitude of triple gauge-boson couplings are presented. The observation of the longitudinal helicity component of the W-boson spin, related to the mechanism of electroweak symmetry breaking, is described together with the techniques used to probe the CP and CPT symmetries in the W-boson system. A discussion on the intricacies of the measurement of the mass of the W boson, whose knowledge is indispensable to test the internal consistency of the Standard Model and estimate the mass of the Higgs boson, concludes this review.

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Introduction

The mission of the LEP project was to further the understanding of the Standard Model of the electroweak interactions, and the study of W bosons is a unique tool to meet this challenge. The study of W-boson pair-production probes two cornerstones of the Standard Model, namely the existence of the coupling between Z and W bosons and of the longitudinal helicity component of the W-boson spin. The mass of the W boson, m_{W}, is a free parameter of the Standard Model and its measurement is therefore indispensable. Precise knowledge of m_{W} allows, through the mechanism of radiative corrections, to test the internal consistency of the Standard Model, as described elsewhere in this Report. In addition, it provides hints on the mass of the yet-unobserved Higgs boson.

As from 1996, the LEP centre-of-mass energy, $\sqrt{s}$, was steadily increased above the W-boson pair-production threshold of $2m_{\mathrm{W}} \approx 161$ GeV, opening a window on W-boson physics at LEP. By the machine shut-down in the year 2000, a total of about 2.7 fb^{-1} of integrated luminosity had been delivered to the four detectors, and around 50 000 events with W-boson pairs were on tape.

W-boson pair-production proceeds through t-channel neutrino exchange and s-channel annihilation with the mediation of either a photon or a Z boson, as shown in Figure 1. The s-channel diagrams are sensitive to the triple gauge-boson couplings (TGCs) γWW and ZWW [1, 2]. More than a thousand W bosons were also single-produced at LEP [2, 3]. This is a particular case of the generic four-fermion process $\mathrm{e^+e^-} \rightarrow \mathrm{e^+}\nu_{\mathrm{e}}\mathrm{f\bar{f}'}$[1], described by diagrams such as those depicted in Figure 2.

The identification of events from W-boson pair-production is discussed in the following sections, together with the determination of the cross section of this process and of W-boson branching fractions. Results from the study of single W-boson production are also presented. Details are then given on the investigation of W-boson polarisation and TGCs. Finally, the measurement of m_{W} is described. Additional details on those topics and a comprehensive list of references can be found in Reference [4]. At the time of writing, a few of these results are still preliminary. Nonetheless, as complex data analyses are nearing conclusion, final results are expected not to show large differences with those discussed below.

W-boson pair-production

W bosons decay either into hadrons or into a charged lepton and a neutrino, with branching fractions of 67.5% and 32.5%, respectively. Therefore, W-boson pair-production is observed in three different topologies:

- "Fully-hadronic events", for 45.5% of the pairs, in which both W bosons decay into hadrons with a signature of four hadronic jets in the detectors;

- "Semi-leptonic events", for 43.9% of the pairs, in which only one W boson decays into hadrons, resulting in events with two hadronic jets, a lepton and missing energy and momentum due to an undetected neutrino;

- "Fully-leptonic events", for 10.6% of the pairs, in which both W bosons decay into leptons with a signature of just two charged leptons and large missing energy and momentum carried away by the two neutrinos.

[1]Unless specified otherwise, charge conjugate processes are implied throughout this Article.

The LEP collaborations developed techniques to select these events with the highest possible efficiency, while suppressing the competing background from other Standard Model processes [5].

Fully-hadronic events are selected with multivariate analyses which rely on event-shape information discriminating four-jet events from two-jet events and on variables quantifying the compatibility of the event kinematics with the production of two W bosons. Three pairings of four jets into two W bosons are possible and the one which best fits the W-boson pair-production hypothesis is retained. Efficiencies of about 80% are reached, for a residual background of about 15%, mostly due to four-jet events originating from higher-order contributions to the process $e^+e^- \rightarrow q\bar{q}$.

Semi-leptonic events are selected by requiring the invariant masses of the two-jet and lepton-neutrino systems to be compatible with m_W. The neutrino four-momentum is deduced from the measured jet and lepton momenta by imposing energy-momentum conservation. Selection criteria on the energy of the charged lepton, the transverse momentum of the event and the direction of the missing momentum reduce the background from $e^+e^- \rightarrow q\bar{q}$ events containing leptons and from four-fermion processes. Efficiencies between 50% and 90% are achieved, the lower values corresponding to tau leptons, which are more complex to reconstruct due to their hadronic decays and the presence of additional neutrinos. The background contamination varies from less than 5% for muons up to 20% for tau leptons and is due to two- and four-fermion processes.

Fully-leptonic events are tagged by the presence of two high-energy charged leptons and large missing energy and momentum. Background from the $e^+e^- \rightarrow \ell^+\ell^-\gamma$ process, where the initial-state-radiation photon escapes undetected along the beam pipe, is reduced by requiring events with large transverse momentum and a missing momentum pointing away from the beam axis. Selection efficiencies vary from 30% to 70%, depending on the lepton flavour, the lowest values corresponding to tau leptons. The background contamination varies between 15% and 30% and is due to two- and four-fermion processes.

In total, around 40 000 W-boson pairs are selected by the four collaborations and the cross sections for W-boson pair-production are measured [5]. The combined results [4] are presented in Figure 3 as a function of $\sqrt{s}$. These results establish the existence of the ZWW coupling, as a much higher cross section would characterise its absence [6]. The measurements are in excellent agreement with the Standard Model predictions [7] as quantified by the ratio, $\mathcal{R}^{WW}$, of the measured, σ_{WW}^{meas}, and the expected, σ_{WW}^{theo}, cross sections:

$$\mathcal{R}^{WW} = \frac{\sigma_{WW}^{meas}}{\sigma_{WW}^{theo}} = 0.997 \pm 0.010.$$

The uncertainty on $\mathcal{R}^{WW}$ receives equal contributions from statistical and systematic uncertainties. The latter are mainly due to the description of QCD processes in both the signal and background modelling.

Low values of $\mathcal{R}^{WW}$ were initially observed, calling for the present improved description of this process [2]. The so-called leading-pole and double-pole approximations were developed to take into account the exchange of a virtual photon between the particles involved in the process. In addition, an improved treatment of initial-state and final-state radiation of photons was also devised. These achievements allowed a reduction of the uncertainty on σ_{WW}^{theo} to the current level of 0.5% [2, 7].

The branching fractions of W bosons are derived from the number of events measured in the different channels [4, 5]. First, the branching fractions into the three different lepton families are determined, without the assumption of lepton universality, as:

$$\begin{aligned}
\mathrm{Br}(\mathrm{W} \to \mathrm{e}\bar{\nu}_{\mathrm{e}}) &= 10.59 \pm 0.17\% \\
\mathrm{Br}(\mathrm{W} \to \mu\bar{\nu}_{\mu}) &= 10.55 \pm 0.16\% \\
\mathrm{Br}(\mathrm{W} \to \tau\bar{\nu}_{\tau}) &= 11.20 \pm 0.22\%.
\end{aligned}$$

The three values are compatible, and assuming lepton universality the branching fraction into hadrons is derived as:

$$\mathrm{Br}(\mathrm{W} \to \mathrm{q}\bar{\mathrm{q}}') = 67.77 \pm 0.28\%.$$

These results are in agreement with the Standard Model predictions. The branching fraction of W bosons into hadrons depends on the six elements $|V_{\mathrm{qq}'}|$ of the Cabibbo-Kobayashi-Maskawa matrix not involving top quarks. LEP measurements provide an estimate of the less-known $|V_{\mathrm{cs}}|$ element as [4]:

$$|V_{\mathrm{cs}}| = 0.989 \pm 0.014.$$

Single W-boson production

The gradual increase of the LEP centre-of-mass energy from 130 GeV up to 209 GeV provided unique conditions to search for manifestations of New Physics beyond the Standard Model. The production of particles predicted by Supersymmetry, for example, would result in striking signatures, such as events with two hadronic jets and large missing energy, due to the production of weakly-interacting, and hence undetected, neutralinos. Surprisingly, such events were found in the LEP data. However, with an invariant mass of the hadronic system close to m_{W} they were ascribed to hadronic decays of W bosons single-produced through the process $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{e}^-\bar{\nu}_{\mathrm{e}}$, rather than to a discovery of Supersymmetry. This process is described by Feynman diagrams like those presented in Figure 2, where the electrons escape detection as they are scattered inside, or close to, the beam pipe. The other signature of single W-boson production is a single charged-lepton in an otherwise empty event. After the first observation of this process [8], around 700 events were selected by the four LEP collaborations [9, 10]. Figure 4 presents the results of a combination of the measured cross sections [4]. A good agreement with the Standard Model predictions [11] is observed, as quantified by the ratio:

$$\mathcal{R}^{\mathrm{We}\nu} = \frac{\sigma^{\mathrm{meas}}_{\mathrm{We}\nu}}{\sigma^{\mathrm{theo}}_{\mathrm{We}\nu}} = 0.978 \pm 0.080,$$

where the uncertainty is mainly statistical. The calculation of $\sigma^{\mathrm{theo}}_{\mathrm{We}\nu}$ is made difficult by the low-angle scattering of the final-state electron and is assigned an uncertainty of 5% [2].

As shown in Figure 2, single W-boson production is sensitive to the γWW coupling and hence to the electromagnetic properties of W bosons. The W-boson magnetic dipole moment, μ_{W}, and electric quadrupole moment, q_{W}, are written as [12]:

$$\mu_{\mathrm{W}} = \frac{e}{2m_{\mathrm{W}}}(1 + \kappa_\gamma + \lambda_\gamma) \qquad q_{\mathrm{W}} = -\frac{e}{m^2_{\mathrm{W}}}(\kappa_\gamma - \lambda_\gamma), \tag{1}$$

where e is the electron charge and the parameters κ_γ and λ_γ describe the coupling of photons and W bosons. In the Standard Model, $\kappa_\gamma = 1$ and $\lambda_\gamma = 0$. Higher-order contributions are well below the statistical precision of LEP [13] data. As an example, a fit to the measured cross section of single W-boson production yields [10]:

$$\kappa_\gamma = 1.12 \pm 0.11,$$

in agreement with the Standard Model predictions. The uncertainty is in equal parts statistical and systematic, the latter being mostly due to the control of signal modelling and instrumental effects.

W-boson polarisation

The spin of W bosons has a transverse and a longitudinal helicity component. The measurement of W-boson polarisation is of particular interest since the longitudinal helicity component arises from the mechanism of electroweak symmetry breaking which gives the W boson its non-vanishing mass. Moreover, a comparison of the helicity fractions of the W^- and W^+ bosons allows a test of CP conservation.

The fractions of the three helicity states of W bosons produced in e^+e^- collisions are a function of both $\sqrt{s}$ and the cosine of the W^- production angle with respect to the electron beam, $\cos\theta_{W^-}$. For the data sample under investigation, Monte Carlo programs [14] predict a longitudinal polarisation of 24%.

The polarisation of pair-produced W bosons is probed by reconstructing the direction in which their decay products are emitted. The experimental analyses are restricted to semi-leptonic events, where the charge of the lepton defines the charge of the W bosons. Denoting the fraction of the helicity states -1, $+1$ and 0 of W^- bosons as f_-, f_+ and f_0, the rest-frame lepton angular spectrum in leptonic W^- decays is given by[2]:

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\cos\theta^*_\ell} = f_-\frac{3}{8}\left(1+\cos\theta^*_\ell\right)^2 + f_+\frac{3}{8}\left(1-\cos\theta^*_\ell\right)^2 + f_0\frac{3}{4}\sin^2\theta^*_\ell. \tag{2}$$

As the quark charge is difficult to reconstruct, the rest-frame angular spectrum in hadronic decays is folded as:

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}|\cos\theta^*_\mathrm{q}|} = f_\pm\frac{3}{4}\left(1+|\cos\theta^*_\mathrm{q}|\right)^2 + f_0\frac{3}{2}\left(1-|\cos\theta^*_\mathrm{q}|\right)^2, \tag{3}$$

where $f_\pm = f_+ + f_-$.

The L3 collaboration performed a fit of Equations (2) and (3) to about 2 000 semi-leptonic events [15]. As shown in Figure 5a, a fit without longitudinal polarisation fails to describe the data. A fit with the three helicity components measures the fraction of longitudinal polarisation to be in agreement with the predictions, with a value:

$$f_0 = 21.8 \pm 3.1\%,$$

where the uncertainty is mainly statistical. The helicity fractions are also measured in four different bins of $\cos\theta_{W^-}$. A good agreement with the predictions is found, as shown in Figure 5b. CP conservation is verified by separately measuring the helicity fractions for W^+ and W^- bosons, which are found to be in agreement, with a statistical accuracy of about 30%.

The OPAL collaboration determined the helicity fractions through the investigation of the W-boson spin-density matrix [16]. The elements of this matrix are defined as [17]:

$$\rho^{W^-}_{\tau\tau'}\left(s,\cos\theta_{W^-}\right) = \frac{\sum_{\lambda,\lambda'} F^{(\lambda,\lambda')}_\tau \left(F^{(\lambda,\lambda')}_{\tau'}\right)^\star}{\sum_{\lambda,\lambda',\tau}\left|F^{(\lambda,\lambda')}_\tau\right|^2},$$

where $F^{(\lambda,\lambda')}_\tau$ is the helicity amplitude to produce a W^- boson with helicity τ from an electron with helicity λ and a positron with helicity λ'. The spin-density matrix is a Hermitian matrix

[2]Assuming CP invariance, f_-, f_+ and f_0 also represent the fractions of the helicity states $+1$, -1 and 0 of W^+ bosons, respectively.

with unit trace described by eight free parameters. The ρ_{++}, ρ_{--} and ρ_{00} diagonal elements correspond to the fractions f_+, f_- and f_0, respectively. The $\rho_{\tau\tau'}$ elements are derived from the measurement of the projection operators $\Lambda_{\tau\tau'}$. These are known functions of the polar and azimuthal rest-frame angles of the final-state fermions and project the differential cross section for W-boson pair-production onto the $\rho_{\tau\tau'}$ elements [18]. By studying a sample of about 4 000 semi-leptonic events, the value:

$$f_0 = 23.9 \pm 2.4\%$$

is obtained, where the uncertainty is mainly statistical. Compatible preliminary results were also reported by the DELPHI Collaboration [19].

CP invariance implies $\rho^{\mathrm{W}^-}_{\tau\tau'} = \rho^{\mathrm{W}^+}_{-\tau-\tau'}$ [20]. Introducing the pseudo time-reversal operator $\hat{\mathrm{T}}$, which transforms the helicity amplitudes into their complex conjugates and is equivalent to the T operator at tree level [21], CP$\hat{\mathrm{T}}$ invariance implies $\rho^{\mathrm{W}^-}_{\tau\tau'} = \left(\rho^{\mathrm{W}^+}_{-\tau-\tau'}\right)^\star$. Therefore, at tree level, only the imaginary parts of the $\rho_{\tau\tau'}$ elements are sensitive to possible CP violation. By introducing the cross sections:

$$\sigma^{\mathrm{W}^\pm}_{\tau\tau'} = \int_{-1}^{+1} \Im\left\{\rho^{\mathrm{W}^\pm}_{\tau\tau'}\right\} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_{\mathrm{W}^-}} \mathrm{d}\cos\theta_{\mathrm{W}^-},$$

three quantities sensitive to tree-level CP violation are formed as:

$$\Delta^{\mathrm{CP}}_{+-} = \sigma^{\mathrm{W}^-}_{+-} - \sigma^{\mathrm{W}^+}_{-+} \qquad \Delta^{\mathrm{CP}}_{+0} = \sigma^{\mathrm{W}^-}_{+0} - \sigma^{\mathrm{W}^+}_{-0} \qquad \Delta^{\mathrm{CP}}_{-0} = \sigma^{\mathrm{W}^-}_{-0} - \sigma^{\mathrm{W}^+}_{+0},$$

as well as three quantities sensitive to loop effects:

$$\Delta^{\mathrm{CP}\hat{\mathrm{T}}}_{+-} = \sigma^{\mathrm{W}^-}_{+-} + \sigma^{\mathrm{W}^+}_{-+} \qquad \Delta^{\mathrm{CP}\hat{\mathrm{T}}}_{+0} = \sigma^{\mathrm{W}^-}_{+0} + \sigma^{\mathrm{W}^+}_{-0} \qquad \Delta^{\mathrm{CP}\hat{\mathrm{T}}}_{-0} = \sigma^{\mathrm{W}^-}_{-0} + \sigma^{\mathrm{W}^+}_{+0}.$$

The measured values of all these quantities are compatible with zero within a statistical accuracy of about 15% and no effects of CP violation are observed [16]. Compatible preliminary results were also reported by the L3 Collaboration [22].

Triple gauge-boson-couplings

The most general form for the VWW vertex, with V denoting either a photon or a Z boson, is described by the effective Lagrangian [21, 23]:

$$\begin{aligned} i\mathcal{L}^{V\mathrm{WW}}_{\mathrm{eff}} = \; & g_{V\mathrm{WW}}\Big[g_1^V V^\mu\left(W^-_{\mu\nu}W^{+\nu} - W^+_{\mu\nu}W^{-\nu}\right) \\ & +\kappa_V\, W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{m_{\mathrm{W}}^2} V^{\mu\nu}W^{+\rho}_\nu W^-_{\rho\mu} \\ & +ig_5^V \varepsilon_{\mu\nu\rho\sigma}\Big((\partial^\rho W^{-\mu})W^{+\nu} - W^{-\mu}(\partial^\rho W^{+\nu})\Big)V^\sigma \\ & +ig_4^V W^+_\mu W^-_\nu(\partial^\mu V^\nu + \partial^\nu V^\mu) \\ & -\frac{\tilde{\kappa}_V}{2}W^-_\mu W^+_\nu \varepsilon^{\mu\nu\rho\sigma}V_{\rho\sigma} - \frac{\tilde{\lambda}_V}{2m_{\mathrm{W}}^2} W^-_{\rho\mu}W^{+\mu}{}_\nu \varepsilon^{\nu\rho\alpha\beta}V_{\alpha\beta}\Big], \end{aligned} \qquad (4)$$

where $F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu$. Once the overall couplings are defined as $g_{\gamma\mathrm{WW}} = e$ and $g_{\mathrm{ZWW}} = e\cot\theta_w$, where θ_w is the weak mixing-angle, seven complex parameters describe the ZWW vertex and seven more describe the γWW vertex. These are too many to be measured simultaneously, and some hypotheses are introduced. First, the CP-violating parameters g_4^V, $\tilde{\kappa}_V$ and

$\tilde{\lambda}_V$ are discarded, as supported by the tests of CP conservation discussed above. In addition, electromagnetic gauge invariance is assumed, fixing $g_1^\gamma = 1$ and $g_5^Z = 0$. The remaining five couplings g_1^Z, κ_γ, κ_Z, λ_γ, λ_Z are assumed to be real. Their Standard Model tree-level values are $g_1^Z = \kappa_\gamma = \kappa_Z = 1$ and $\lambda_\gamma = \lambda_Z = 0$.

Custodial SU(2) symmetry [13, 18, 23] implies $\kappa_Z = g_1^Z - \tan\theta_w(\kappa_\gamma - 1)$ and $\lambda_Z = \lambda_\gamma$ and reduces the parametrisation of TGCs to three quantities: g_1^Z, κ_γ and λ_γ. As presented in Equation (1), κ_γ and λ_γ are related to the W-boson electromagnetic properties [12].

The differential cross section of W-boson pair-production exhibits a strong dependence on g_1^Z, κ_γ and λ_γ. For unpolarised initial states, summing over the final-state fermion helicities, fixing m_W and neglecting photon radiation, five angles describe completely the phase space of W-boson pair-production, Ω, and are used for the TGC determination. In addition to θ_{W^-}, these are the rest-frame polar and azimuthal decay angles of the fermions from the W^- decays and of the anti-fermions from the W^+ decays.

For semi-leptonic events the determination of the charge of the W bosons, crucial for the reconstruction of θ_{W^-}, is accurate. These events also allow the identification of the fermion and anti-fermion in the W-boson leptonic decay. On the other side of the event, no attempts to identify the fermion and the anti-fermion in W-boson hadronic decays are usually made and folded angular distributions are considered.

For fully-hadronic events, jet-charge techniques result in a satisfactory tagging of the W-boson charge and allow to reconstruct θ_{W^-}. Folded distributions are used for all rest-frame decay angles.

The largest sensitivity to TGCs comes from $\cos\theta_{W^-}$. Figure 6 compares its distributions, as observed by the OPAL collaboration, with the predictions for the Standard Model value $\lambda_\gamma = 0$ and for $\lambda_\gamma = \pm 0.5$.

A method the determination of TGCs is to fit to the data the five-dimensional differential cross section obtained by re-weighting Monte Carlo events as a function of g_1^Z, κ_γ and λ_γ [24]. Fits to each of the tree couplings are performed as well as simultaneous fits to two or three couplings. As an example, Figure 7 presents the results of the fits for g_1^Z and κ_γ.

TGCs are also determined with an "optimal observable analysis" [25]. As the Lagrangian of Equation (4) is linear in the TGCs, α_i, the differential cross section for W-boson pair-production is a second-order polynomial function:

$$\frac{\mathrm{d}\sigma(\Omega, \alpha_i)}{\mathrm{d}\Omega} = S^{(0)}(\Omega) + \sum_i \alpha_i S_i^{(1)}(\Omega) + \sum_{i,j} \alpha_i \alpha_j S_{ij}^{(2)}(\Omega),$$

where the functions $S^{(0)}$, $S_i^{(1)}$ and $S_{ij}^{(2)}$ are known. All the information on the TGCs is then summarised by the observables:

$$\begin{aligned} \mathcal{O}_i^{(1)} &= S_i^{(1)}(\Omega)/S^{(0)}(\Omega) \\ \mathcal{O}_i^{(2)} &= S_{ii}^{(2)}(\Omega)/S^{(0)}(\Omega) \\ \mathcal{O}_{ij}^{(2)} = \mathcal{O}_{ji}^{(2)} &= S_{ij}^{(2)}(\Omega)/S^{(0)}(\Omega), \end{aligned}$$

which are reconstructed from data and fit to determine the TGCs [26].

Compatible results are found by all LEP collaborations and their preliminary combination gives [4]:

$$g_1^Z = 0.991^{+0.022}_{-0.021} \quad \kappa_\gamma = 0.984^{+0.042}_{-0.047} \quad \lambda_\gamma = -0.016^{+0.021}_{-0.023},$$

in agreement with the Standard Model prediction. The uncertainties are in equal part statistical and systematic. The latter follows from the theoretical uncertainties on the description of

the differential cross sections of W-boson pair-production [2]. Results from two- and three-dimensional fits are also in agreement with the Standard Model predictions.

These results also include information from a partial reconstruction of fully-leptonic events and from single W-boson production and single-photon production. The last phenomenon is mostly due to the radiation of a photon in the initial state of the process $\mathrm{e^+e^-} \to \nu\bar{\nu}$. However, it also receives a small contribution from the $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\gamma$ process where the photon is produced through W-boson fusion in a γWW vertex. Semi-leptonic and fully-hadronic events from W-boson pair-production are largely more sensitive than these other processes. In particular, they are two, ten and five times more sensitive than fully-leptonic events for g_1^Z, κ_γ and λ_γ, respectively, around ten times more sensitive than single W-boson production for λ_γ, and four and twenty times more sensitive than single-photon production for κ_γ and λ_γ, respectively. The only exception is the comparable sensitivity of single W-boson production to κ_γ.

If the W boson were an extended object, such as an ellipsoid of rotation, its average radius R_W would be related to its magnetic dipole moment and hence to the TGC as: $R_\mathrm{W} = (\kappa_\gamma + \lambda_\gamma - 1)/m_\mathrm{W}$ [27]. The measurements [24] indicate that W bosons are point-like particles down to a scale of 10^{-19} m:

$$R_\mathrm{W} = (0.3 \pm 1.9) \times 10^{-19}\ \mathrm{m}.$$

W-boson mass

Early measurements of m_W at LEP were performed with about 10 pb^{-1} of data collected by each experiment at the W-boson pair-production threshold, where the cross section depends strongly on m_W [28]. The combined result reads [4]:

$$m_\mathrm{W}^\mathrm{threshold} = 80.40 \pm 0.21\,\mathrm{GeV},$$

where the uncertainty is mainly statistical.

Higher LEP centre-of-mass energies allow direct reconstruction of W bosons and mass spectra are obtained for fully-hadronic and semi-leptonic events, as shown in Figure 8. Three techniques were developed to extract m_W from these spectra or from related quantities. A first technique is to fit the reconstructed m_W spectrum with a Breit-Wigner function, using detector resolutions obtained from Monte Carlo simulations. A second technique is to compare directly Monte Carlo simulations which include all known detector effects and physical processes to the data. Re-weighting techniques are used to obtain simulations which are a function of m_W. A fit indicates the value of m_W which best describes the data. A last technique convolves all known detector and physical processes to obtain a probability-density function for m_W and a likelihood analysis of the data indicates the most probable value of m_W. A combination [4] of the preliminary results of the four experiments [29] yields:

$$m_\mathrm{W}^{\mathrm{qq}\ell\nu} = 80.411 \pm 0.032\,(\mathrm{stat.}) \pm 0.030\,(\mathrm{syst.})\,\mathrm{GeV}$$

$$m_\mathrm{W}^{\mathrm{qqqq}} = 80.420 \pm 0.035\,(\mathrm{stat.}) \pm 0.101\,(\mathrm{syst.})\,\mathrm{GeV}.$$

for semi-leptonic and fully-hadronic final states, respectively, with a correlation coefficient of 0.18. Their combination is:

$$m_\mathrm{W} = 80.412 \pm 0.029\,(\mathrm{stat.}) \pm 0.031\,(\mathrm{syst.})\,\mathrm{GeV}.$$

This value also includes the results from the threshold measurements and information from partial reconstruction of fully-leptonic events [30], which have a large statistical uncertainty

and hence little impact on the combined value. Analysis methods similar to those used for the determination of m_W are also used for the determination of the W-boson width, Γ_W, with the result:

$$\Gamma_W = 2.150 \pm 0.091\,\text{GeV}.$$

	Uncertainties on m_W (MeV)		
Source of systematics	Semi-leptonic	Fully-hadronic	Combined
Bose-Einstein correlations	–	35	3
Colour reconnection	–	90	9
Beam energy	17	17	17
Hadronisation	19	18	18
Detector	14	10	14
Other	8	9	8
Total systematic	31	101	31

Table 1: Sources of systematic uncertainty in the determination of m_W for hadronic and semi-leptonic final states and their combination.

The effects of different sources of systematic uncertainty are listed in Table 1 [4] and discussed in the following. Fully-hadronic events are affected by large systematic uncertainties which are correlated among experiments and are due to Bose-Einstein correlations (BEC) and colour reconnection (CR). Their weight in the combination is therefore only 10%, sizably reducing the statistical power of the analysis. At the time of writing, a challenging program to reduce these uncertainties is in progress.

BEC are responsible for the enhancement of the production of pairs of identical bosons close together in phase space. They were observed in Z-boson decays and understood from quantum-mechanical principles. BEC in W-boson decays were also observed and found to be similar to those of Z-boson decays into light quarks [31]. The presence of BEC between particles originating from decays of different W bosons would modify the kinematics of the final-state particles and affect the correspondence between m_W and the measured jet four-momenta [32, 33]. A large value for these "inter-W" BEC [34] would induce a shift on m_W of 35 MeV [4], assumed as systematic uncertainty in Table 1. In order to reduce this uncertainty, the LEP collaborations have directly measured the amount of inter-W BEC by comparing particle-correlation functions measured in fully-hadronic events with those of four-jet events with no correlation. These are obtained by superimposing the hadronic parts of two different semi-leptonic events [31]. A combination of the results suggests that only $(23 \pm 13)\%$ of the possible large effect is observed in data [4]. This result is compatible with little or no BEC and reduces the present uncertainty on m_W of 35 MeV to about 13 MeV.

The hadronisation of quarks from W-boson decays happens on a scale of 0.1 fm. Hadronic interactions, on the other hand, have a larger characteristic distance of about 1 fm, which means that a substantial cross-talk is possible between hadrons originating from different W bosons. This process, called "colour reconnection", could modify the four-momenta of the observed jets and introduce a shift in the measured value of m_W [33, 35]. The LEP collaborations have performed direct measurements of the extent of CR in fully-hadronic events by comparing particle densities in regions between jets originating from the same W bosons with those in the regions between jets originating from different W bosons. Extreme CR models are excluded [4,

36]. This finding is corroborated by the compatibility of the m_{W} determinations in the semi-leptonic and fully-hadronic channels:

$$\Delta m_{\mathrm{W}} = m_{\mathrm{W}}^{\mathrm{qq}\ell\nu} - m_{\mathrm{W}}^{\mathrm{qqqq}} = -22 \pm 43\,\mathrm{MeV}.$$

This data-driven estimate of CR effects is robust, but results into a large range of possible shifts on m_{W}, reflected by the 90 MeV systematic uncertainty of Table 1. A viable approach to reduce this uncertainty is to make the measurement less sensitive to CR effects. As CR mainly affects inter-jet regions, the LEP collaborations are now modifying their clustering algorithms in order to consider increasingly-narrow hadronic jets for the determination of m_{W}. Only events affected by a small uncertainty on CR will be retained. Preliminary results indicate that this systematic uncertainty can be halved at the price of an increase of 10% of the statistical uncertainty.

The determination of the LEP centre-of-mass energy is a source of a systematic uncertainty which is correlated among all experiments and channels. This follows from the use of $\sqrt{s}$ in kinematic fits for the event reconstruction, resulting into an uncertainty on m_{W} given by $\Delta m_{\mathrm{W}}/m_{\mathrm{W}} = \Delta\sqrt{s}/\sqrt{s}$. Improved calibrations of the LEP centre-of-mass energy [37] will contribute to the reduction of this uncertainty. As a cross check, the LEP experiments have reconstructed the mass of the Z boson, m_{Z}, using $\mathrm{e^+e^-} \rightarrow \mathrm{Z}\gamma$ events [38]. The results depend on $\sqrt{s}$. As they are in agreement with the precision measurement of m_{Z} obtained from scans of the Z resonance, they validate the results of the LEP energy calibration.

Another source of systematic uncertainty which is correlated among experiment and channels, is the modelling of the hadronisation process. As in the case of BEC and CR, possible changes in the phase-space of the hadrons affect the m_{W} reconstruction. Different hadronisation models exist. They were carefully "tuned" to reproduce experimental distributions observed at LEP and at lower $\sqrt{s}$ and included in the Monte Carlo simulations used to measure m_{W}. A comparison of the results obtained by using different hadronisation models results in an uncertainty on m_{W} of 18 MeV.

The high precision of the m_{W} measurement calls for a detailed understanding of detector resolutions and response. Uncertainties on the energy scales of the calorimeters, and the angular determination of leptons and jets directly affect m_{W}. These uncertainties may be sizable, but are not correlated among the experiments and are largely diluted in the combination.

In conclusion, it is expected that the improved control of the systematic uncertainties discussed above will increase the impact of fully-hadronic events in the measurement of m_{W} and will thus reduce the total uncertainty on m_{W} to around 35 MeV.

Summary

The study of W-boson physics at LEP has been a success. The existence of the triple gauge-boson coupling ZWW was established, confirming the non-Abelian structure of the Standard Model of the electroweak interactions. W-boson longitudinal polarisation, which is a consequence of the electroweak symmetry-breaking mechanism, was observed and measured to be in agreement with the Standard Model predictions. Within the available statistical precision, no hints of CP violation in the W-boson system were found.

Several quantities describing fundamental properties of W bosons were measured with an accuracy of a few percent: branching ratios, magnetic dipole moment, electric quadrupole moment and couplings to Z bosons. The mass and the width of the W bosons were measured to be:

$$m_{\mathrm{W}} = 80.412 \pm 0.042\,\mathrm{GeV} \qquad \Gamma_{\mathrm{W}} = 2.150 \pm 0.091\,\mathrm{GeV}.$$

This value of m_W is in agreement with, and improves upon, the measurements from hadron colliders. At the time of writing, challenging studies aim to reduce the uncertainty on m_W to around 35 MeV.

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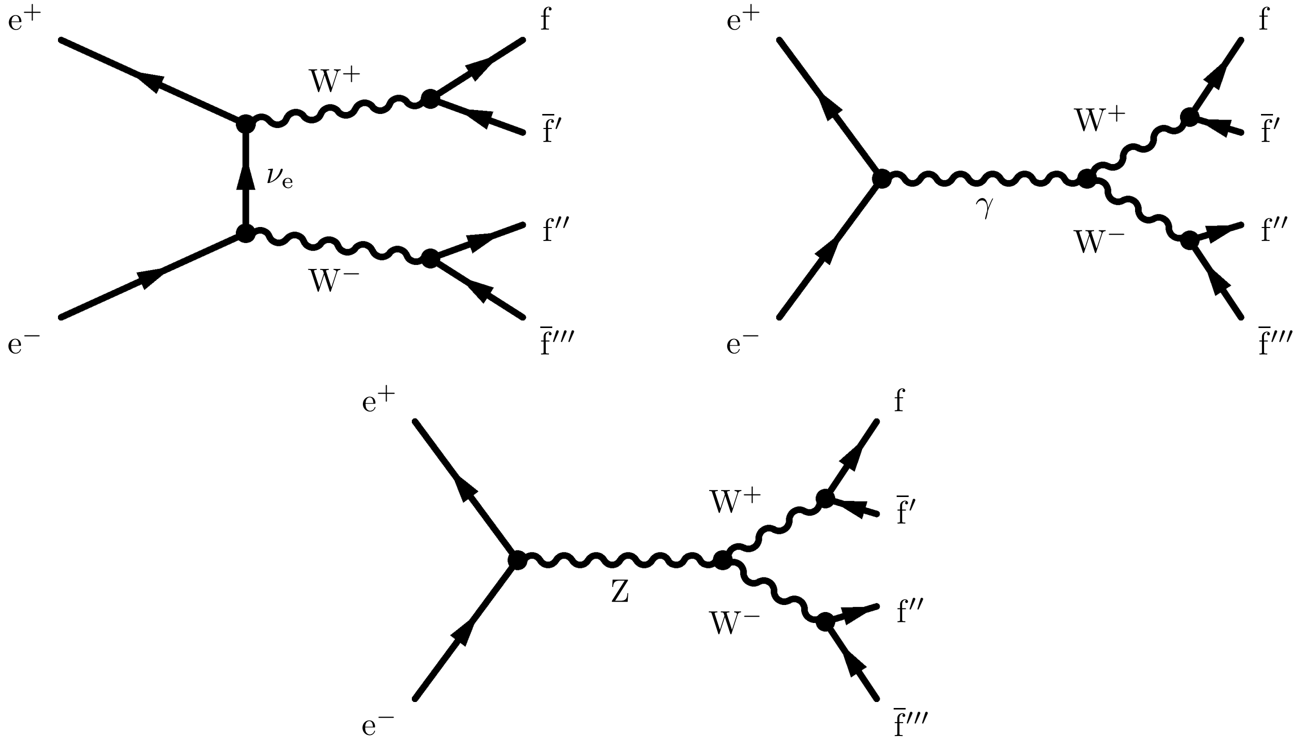

Figure 1: Feynman diagrams describing W-boson pair-production at LEP.

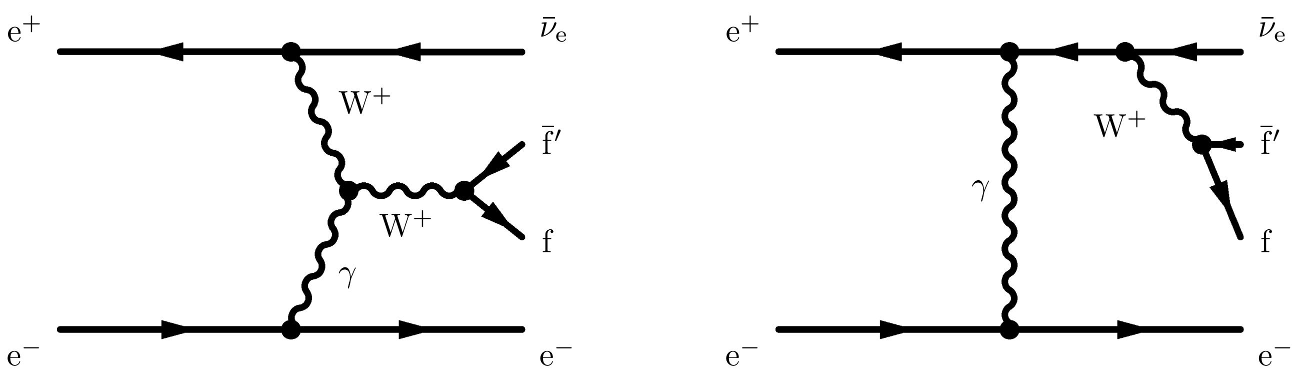

Figure 2: Some of the Feynman diagrams describing single W-boson production at LEP.

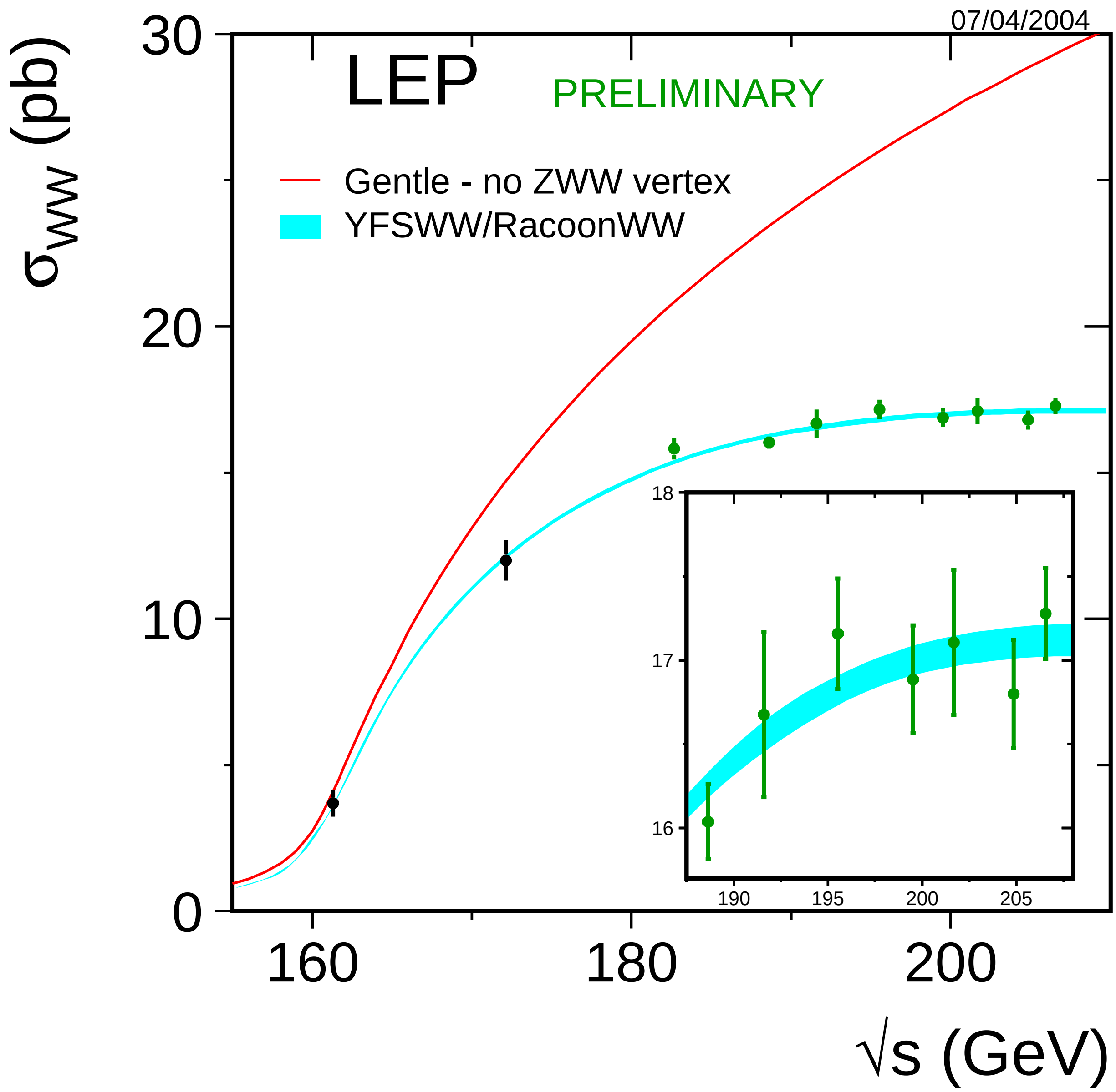

Figure 3: Measurement of the cross section for W-boson pair-production at LEP as a function of $\sqrt{s}$. Values above 180 GeV are still preliminary. Standard Model predictions [7] are indicated by the band, whose width represents the theoretical uncertainty of 0.5% [2, 7]. Predictions in the absence of the ZWW couplings [6] are also shown.

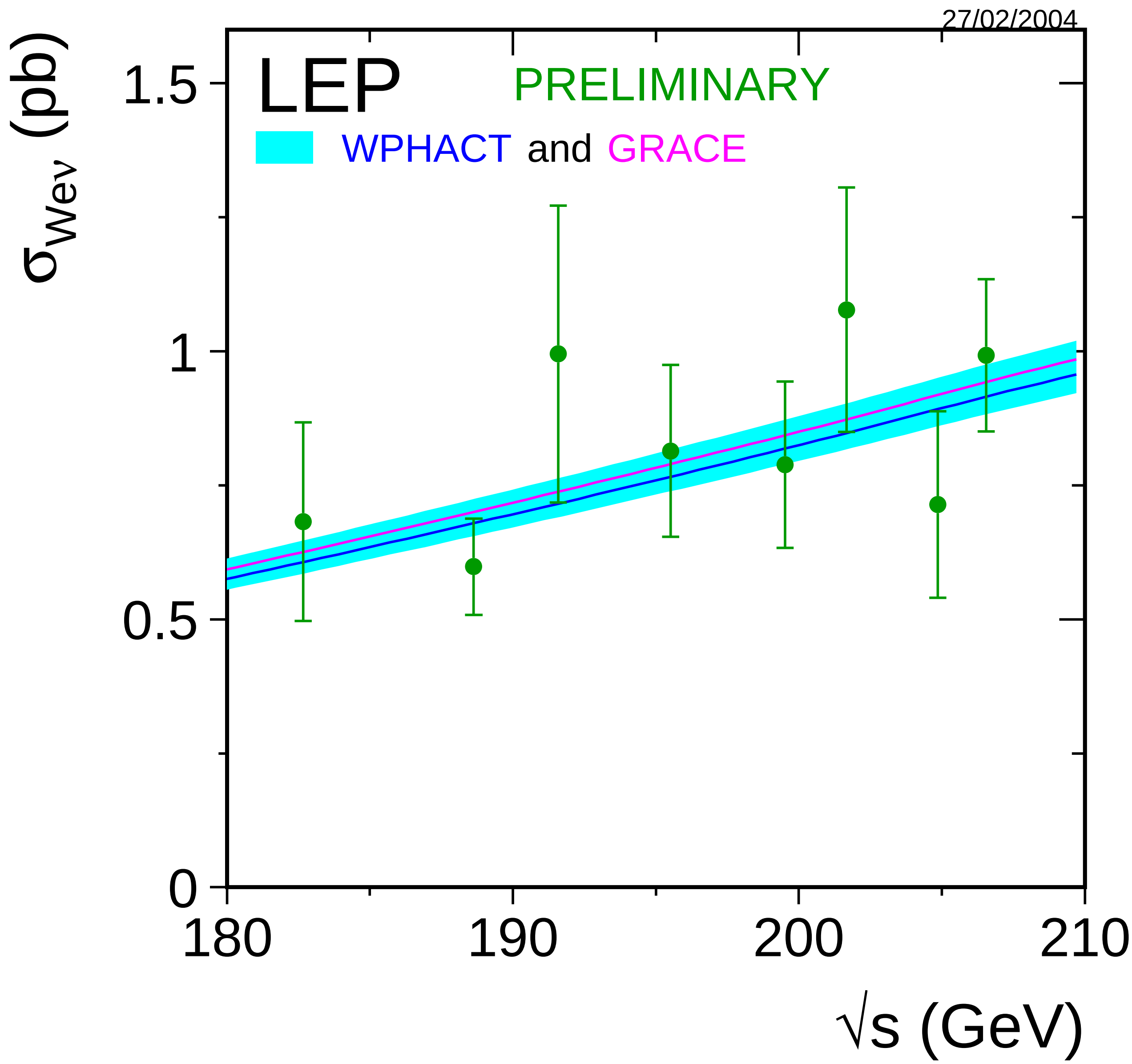

Figure 4: Measurement of the cross section for single W-boson production at LEP as a function of $\sqrt{s}$. Standard Model predictions [11] are indicated by the band, whose width represent the theoretical uncertainty of 5% [2].

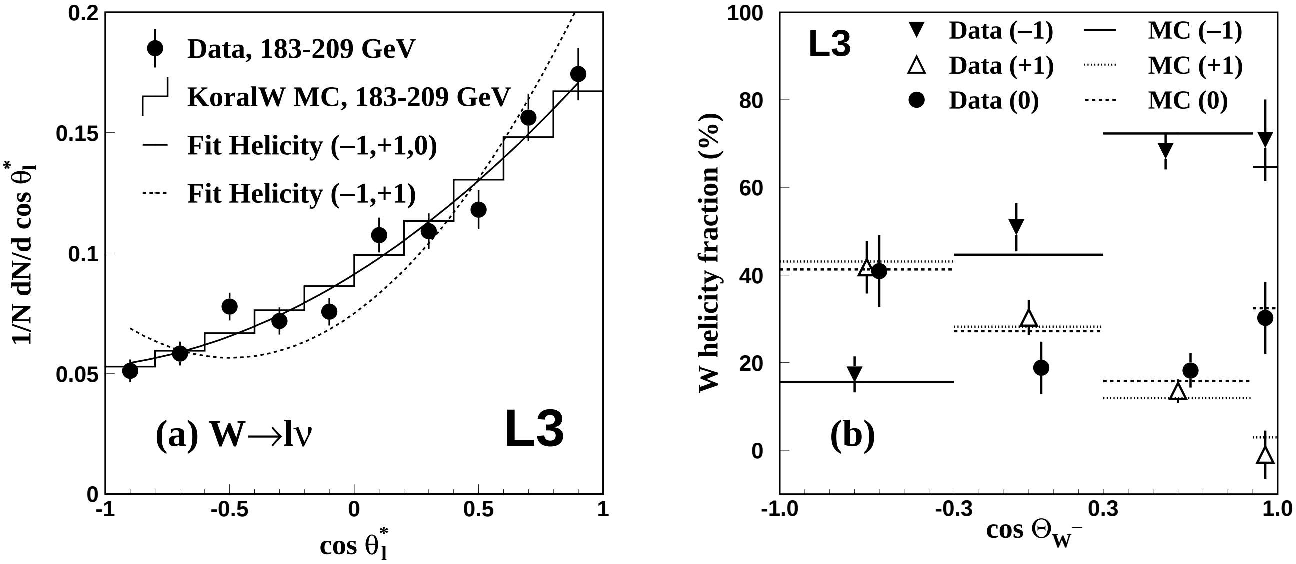

Figure 5: a) Rest-frame lepton angular spectrum observed in data compared with Standard Model Monte Carlo expectations. Results of fits with two and three helicity states are shown. A fit with no longitudinal polarisation fails to describe the data. b) Helicity fractions of W bosons measured as a function of the cosine of the W^- polar angle.

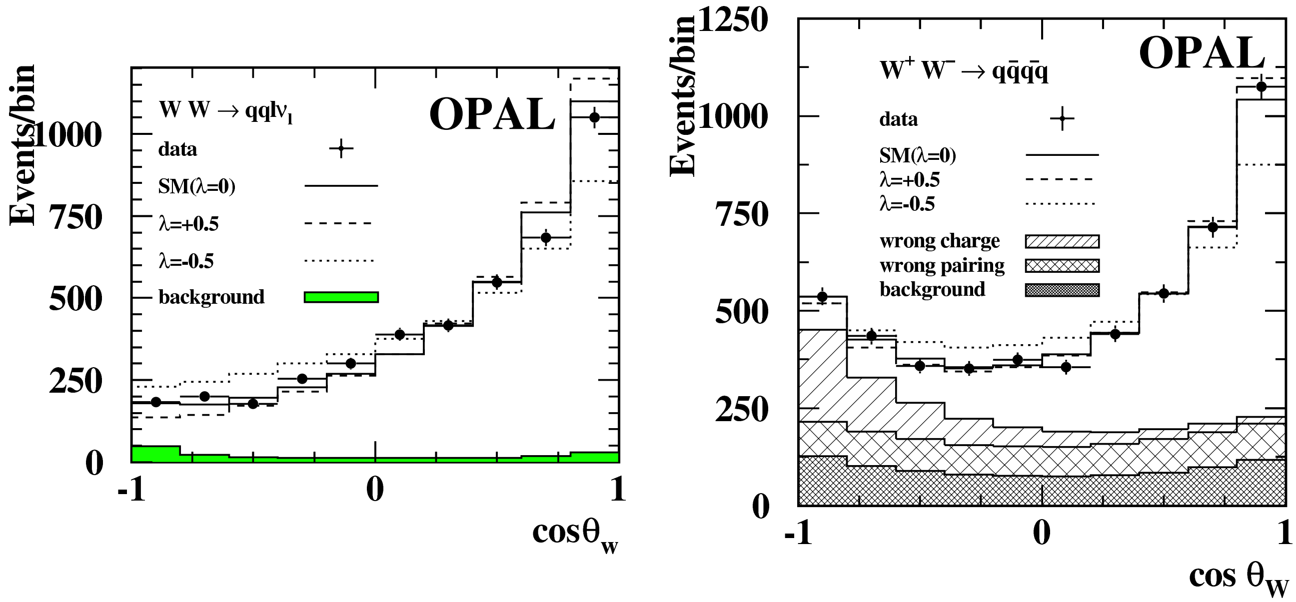

Figure 6: Differential distribution for the cosine of the W^- polar angle for semi-leptonic and fully-hadronic events. Predictions from the Standard Model and in presence of an anomalous value of the coupling λ_γ are also given.

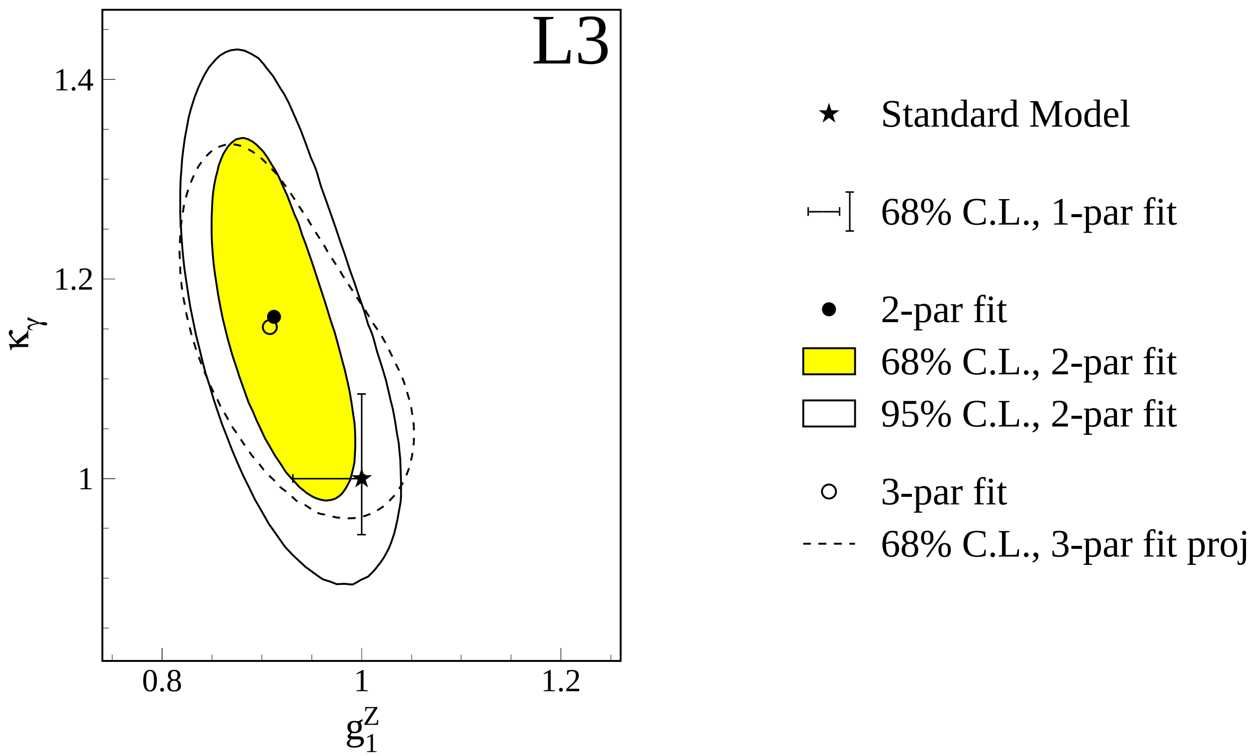

Figure 7: Results of one-, two- and three-dimensional determinations of the couplings g_1^Z and κ_γ.

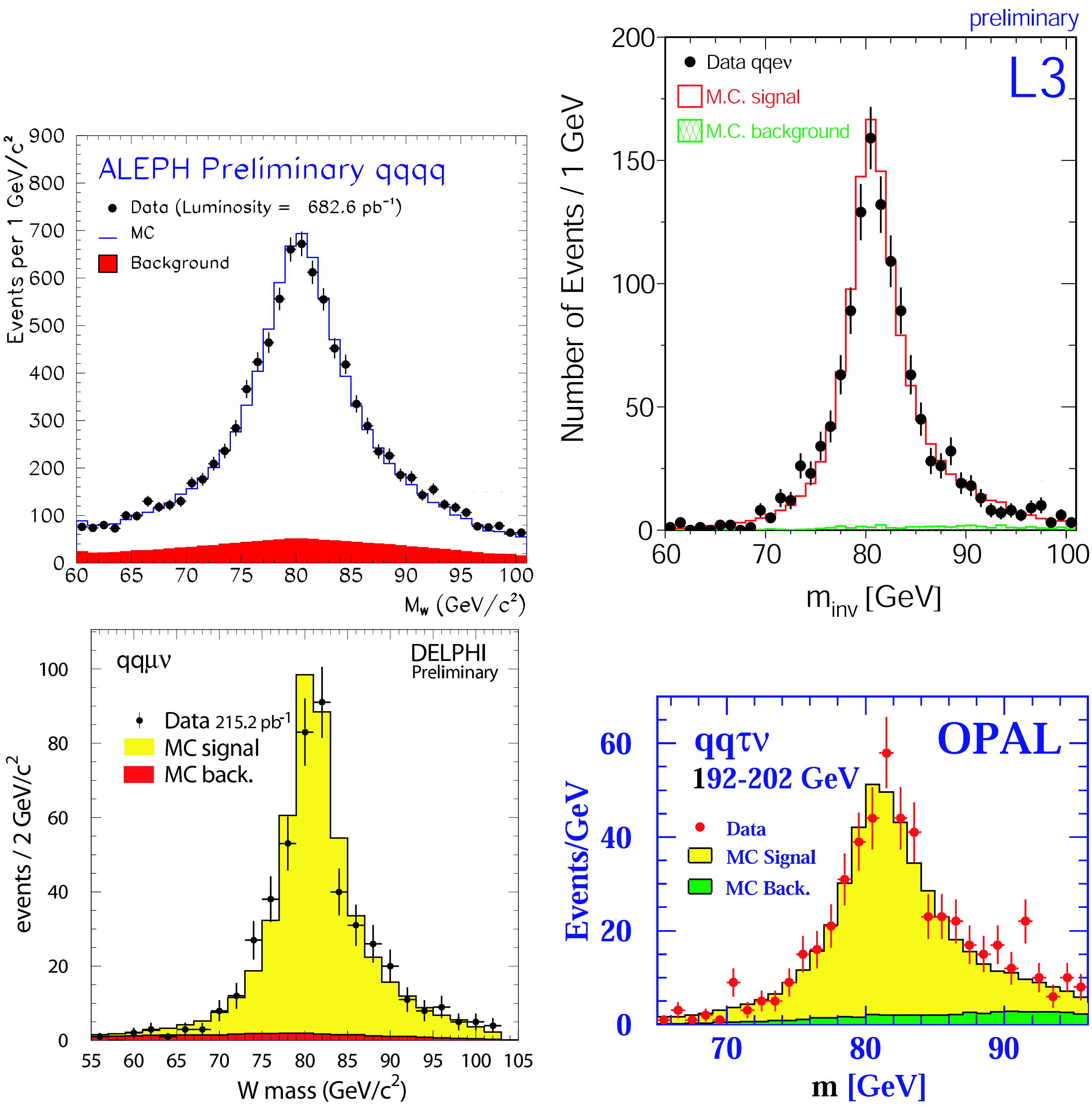

Figure 8: W-boson mass spectra reconstructed for fully-hadronic events and semi-leptonic events with electrons, muons and tau leptons.